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Enhanced MPC Based on Unknown State Estimation and Control Compensation

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Highlights

- An enhanced MPC method for stochastic system with unmeasurable state and non-Gaussian noise is proposed
- The Kalman filter is used to estimate the state and the compensator is calculated without changing the existing MPC controller
- The best compensator parameter is achieved by optimizing the variance of the tracking error
- Stability of the proposed En-MPC method is guaranteed in the mean square sense by restricting the upper bound of the state
- Numerical simulation and sewage treatment process experiment verify the superiority of the proposed method

Enhanced MPC Based on Unknown State Estimation and Control Compensation

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Abstract: The model predictive control (MPC) method is widely used in multivariable process control due to its optimization control nature and easy engineering realization. Aiming at the large control error fluctuations caused by non-Gaussian noise, unmeasurable state, and process disturbance under the MPC method in industrial practice, this paper proposes a novel enhanced MPC (En-MPC) method that uses Kalman filter to estimate unknown states and combines with the state gain matrix for control compensation. Firstly, given the difficulty of measuring some key states of actual industrial processes, the unknown state variables are estimated online through Kalman filter technology; Secondly, the state estimation values are used as the initial value of the prediction model to obtain the future output information of the system, and the open-loop optimization solution is calculated in the finite horizon by solving the optimization objective function. The relationship equation between the state variance and the state gain matrix is established and optimized to obtain the optimal gain matrix, which is multiplied by the state estimation as the output of the compensation controller. Finally, the solution of the basic optimization controller and the solution of the compensator is added together to act as the final control input to the controlled plant. The upper bounds of the state variables of the proposed method are proved by the induction method in the root-mean-square sense, and the stability of the system under the algorithm is demonstrated. Simulations and sewage treatment process data experiment show the effectiveness and practicability of the proposed method.

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1 Introduction

The process of modern industrial production is becoming more complicated, and the scale is getting larger than before. Naturally, it puts forward higher requirements on the control system. When the control requirements of industrial systems develop from equipment-level single-loop control to system-level multi-loop control, the methods such as single-loop PID control cannot guarantee the global performance of multi-variable complicated systems, due to the lack of decoupling ability. When the control objective of an industrial system develops from single regulation control to overall performance optimization, feedback control such as a single PID cannot adjust the dynamic process timely and accurately, and it is even more difficult to achieve global optimization. Therefore, the model predictive control (MPC) method with strong multi-variable and constraint processing capabilities has shown obvious advantages and received more and more attention and applications [1-3]. Originated in the mid-1970s, MPC's basic idea is to solve a finite-time open-loop optimization problem online, based on the sampling information at the current moment, and apply the first set of optimized solutions to the controlled plant, and then repeat the above process at the next moment [4-5]. Compared with PID and other control methods to solve a feedback control law at one time, MPC can better meet the dynamic control performance requirements of the system by solving open-loop optimization problems online and obtaining optimized solutions in the prediction horizon [6].

In practical engineering applications, it is hard to accurately model industrial processes, and nonlinear constrained optimization problems are difficult to solve online in the industrial field. As a result, nonlinear predictive control is still subject to many restrictions and linear predictive control has been the mainstay in practical engineering applications. Obviously, most industrial systems are nonlinear. Even if the dynamic process can be approximated as a linear system, the dynamic process of the system also exhibits nonlinear characteristics due to a series of problems such as constraints on the actuator. Besides, when the model predictive controller is designed, its objective function is only about the mean and variance of the system output or error. This is reasonable when the noise distribution is relatively uniform because the probability distribution can be fully described by the mean and variance at this time [7-10]. However, when encountering various forms of noise in the actual industrial site, the above-mentioned objective function of predictive control cannot capture the statistical characteristics of all process dynamics, and the open-loop optimal solution obtained in the finite horizon is also difficult to satisfy high-performance control requirements. Also, any actual industrial system runs in a dynamic environment. No matter what process, disturbances and uncertain dynamics always exist. If these external disturbances are not monitored and suppressed in time, they will have an adverse effect on the process, in severe cases, and it will also affect the stability and safety of the process operation. Since MPC and other control methods do not directly consider and deal with these process disturbances and uncertain dynamics during controller design, it is difficult to obtain satisfactory control performance when controlled industrial processes exist strong external disturbances and uncertain dynamics, which may lead to a large tracking error of the controlled plant.

Given the problems of the above-mentioned MPC, inspired by the minimum variance control [11] and the Kalman filter technology [12, 13], this paper proposes an enhanced predictive control method to reduce the control error. Based on the existing state-space model and the unmeasurable state estimation, the proposed method mainly includes two parts: the first part is the basic state-space model optimization controller, which is mainly used to calculate the basic control input and ensure the basic control performance of the closed-loop system; the second part is the compensation controller proposed to reduce the fluctuation of output, mainly used to suppress the uncertain dynamics and external disturbance of the process, and enhance the performance of the closed-loop control system. Moreover, the compensator uses Kalman filter technology to calculate the posterior estimation of the unknown state, and the gain matrix of the compensator is updated iteratively through a recursive relationship based on variables which are the uncertain dynamic variance at the current moment. Through inductive reasoning, the upper bound of the system state in the root-mean-square sense is proved. The main differences between the method in this paper and the methods in the existing literatures are as follows:

1) Compared with the neural network based predictive control method in [14, 15], the predictive control in this paper is based on a state-space prediction model, which has the advantage of stability and practicability since the model of the actual industrial plant can be obtained through methods such as subspace identification [16, 17].
2) To improve the control effect, different from adding constraints to the state variables in the objective function of the existing MPC algorithms [18], this paper considers the influence of the state variables on the output and use recursively updated matrix as the gain of the state feedback to reduce the fluctuation of the control error.
3) To reduce the fluctuation of the tracking error, different from the adjustment strategy of predictive control in [19], the method in this paper does not correct the state variables or output of the prediction model but seeks the optimal compensation to correct the input.
4) The proposed enhanced predictive control algorithm, which is based on unknown state estimation and control compensation, only adds compensation to the original predictive control input to reduce the fluctuation of control error, so there is no need to change the existing predictive control structure, leading to the low cost. Therefore, it is easy to implement in engineering practice.

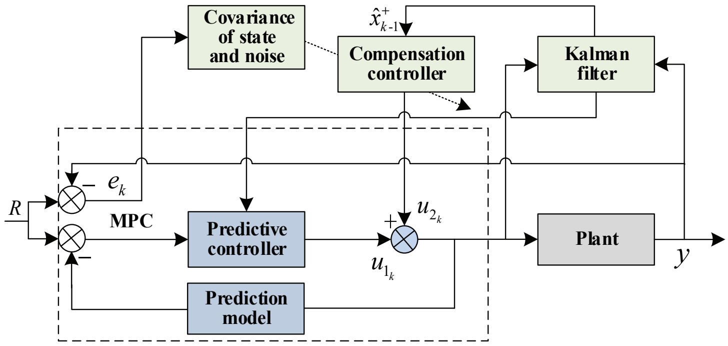

Fig.1. The control architecture of the proposed En-MPC

2 Control Strategy

The strategy structure of the proposed enhanced model predictive control (En-MPC) method is shown in Fig.1. The dashed box is the basic predictive controller. The controlled plant is an actual industrial process that can be approximated as a linear model. Rolling optimization obtains an open-loop solution in a finite horizon by solving a present objective function. Outside the dashed box is the compensation control algorithm proposed to reduce the fluctuation of control error: First, the posterior estimated value of the unknown state variable of the system is obtained through the Kalman filter technology. Then, to reduce the variance of the error, an optimal gain matrix is obtained and multiplied by the estimated value of the state variable as the compensation of the basic predictive control input. Finally, the obtained integrated value of basic control input and compensation input is used as the final control input to act on the controlled plant.

The controlled plant is described as follows:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + B_d w_k \\ y_k = Cx_k + Fv_k \end{cases} \tag{1}$$

where x_k is the state variable and $y_k \in \mathbb{R}^{n_y \times 1}, u_k \in \mathbb{R}^{n_u \times 1}$ represent the output and the input, w_k is the process noise, and v_k stands for the measuring noise, A, B, B_d, C, F are the system matrix of appropriate dimensions.

As shown in Fig.1, the control input is composed of two parts, as shown follows:

$$u_{k\text{-}1} = u_{1_{k\text{-}1}} + u_{2_{k\text{-}1}} \tag{2}$$

where $u_{1_{k-1}}$ is the basic control input of MPC, it can be calculated by the following Eq.(3):

$$u_{1_{k\text{-}1}} = K_{\text{mpc}} E_p(k|k-1) \tag{3}$$

where K_{mpc} is the gain matrix, $E_p(k|k-1)$ is an integrated error and calculated by Eq.(20) in Section 3.2.

The second part $u_{2_{k-1}}$ of the control input is the solution of the enhanced compensation controller, as follows:

$$u_{2_{k\text{-}1}} = K_{1_{k\text{-}1}} \hat{x}^+_{k\text{-}1} \tag{4}$$

where $\hat{x}^+_{k\text{-}1}$ is the posterior state variable estimated by Kalman filter and $K_{1_{k\text{-}1}}$ stands for the gain matrix.

Remark 1. Since the main purpose of the proposed method is to reduce the fluctuation of the control error, it can be derived from the theorem about the variance [20] that the variance of the error is equal to the variance of the control output. Then, minimizing the variance of error can be converted into minimizing the variance of state controlled system. Therefore, reducing the fluctuation of the control error can be converted to optimizing the variance of the state through adding state feedback. □

Remark 2. Due to some state variables cannot be directly measured, the compensator first employs Kalman filter to obtain the estimation of the state, and then multiplies with the gain matrix as the output of the compensator. Therefore, the gain matrix in Eq.(4) is the focus of the compensator which is used to reduce the variance of the state variable and further reduce the tracking error fluctuation, by adding the feedback of the state estimation value. □

Remark 3. In general, the stability of the predictive control is mainly guaranteed by extending the finite prediction horizon to the infinite horizon, but this makes the numerical solution difficult to calculate in the industrial field. Another method is the terminal constraint predictive control, which forces the terminal state to return to the equilibrium point by adding constraints to the open-loop optimization but adding state constraints to the terminal artificially will greatly increase the number of online calculations. Because the state space predictive control is adopted in this paper, the stability of the proposed hybrid control method will be proved by recursive analysis of the boundedness of state variables. □

3 Control Algorithm

3.1 Prediction model

The main function of the prediction model is to predict the future state. From this point of view, the neural networks models with more complex forms may have better effects in simulation. However, considering the complexity of the neural network model and the limitation of the high-throughput computing capacity in the industrial field, this paper adopts the state space prediction model, which is not only simple in structure, low in calculation cost, but also stable. To obtain the analytical formula of the prediction model, we give the following assumptions.

Assumption 1. Supposing that after k sampling time, the process noise w_k keeps constant, which is $w_{k+i|k}=0$, $i=1,\cdots,p-1$, and the measuring noise v_k keeps constant, which is $v_{k+i|k}=0$, where p is the prediction horizon, m is the control horizon. □

Remark 4. In the actual process, the noise and disturbances have a large randomness. If the noise and disturbance model is introduced into the prediction model, it may cause the phenomenon of overfitting and a larger control error. Therefore, the prediction model in this article only considers the main process of the controlled plant [19]. □

Under Assumption 1, expand the predicted value of the system as a combination of known variables. Supposing the state of the discrete system described by Eq.(1) is x_k at k sampling time, then the states after two sampling time are shown in follows:

$$\begin{aligned} x_{k+1|k} &= Ax_k + Bu_k + B_d w_k \\ x_{k+2|k} &= Ax_{k+1|k} + Bu_{k+1} + B_d w_{k+1} = A^2 x_k + ABu_k + Bu_{k+1} + AB_d w_k \end{aligned} \tag{5}$$

By Assumption 1, after k sampling time in the prediction horizon, the process noise w_k keeps constant, which is $w_{k+i|k}=0, i=1,\cdots,p-1$, then $x_{k+p|k}$ can be calculated as follows:

$$\begin{aligned} x_{k+p|k} &= Ax_{k+p\text{-}1|k} + Bu_{k+p\text{-}1} + B_d w_{k+p\text{-}1} \\ &= A^p x_k + A^{p\text{-}1}Bu_k + A^{p\text{-}2}Bu_{k+1} + \cdots + A^{p\text{-}m}Bu_{k+m-1} + ABu_{k+p-2} + Bu_{k+p-1} + A^{p\text{-}1}B_d w_k \end{aligned} \tag{6}$$

By Assumption 1, after k sampling time in the prediction horizon, the measuring noise v_k keeps constant, which is $v_{k+i|k}=0, i=1,\cdots,p-1$, then the output $y_{k+1|k}$ between $k+1$ and $k+p$ sampling time are shown as follows:

$$\begin{aligned} y_{k+1|k} &= Cx_{k+1|k} + Fv_{k+1} \\ &= CAx_k + CBu_k + CB_d w_k + Fv_k \\ &\vdots \\ y_{k+p|k} &= Cx_{k+p|k} + Fv_{k+p} \\ &= CA^p x_k + CA^{p\text{-}1}Bu_k + CA^{p\text{-}2}Bu_{k+1} + \cdots + CA^{p\text{-}m}Bu_{k+m-1} + \cdots + CABu_{k+p-2} + CBu_{k+p-1} + CA^{p\text{-}1}B_d w_k + Fv_k \end{aligned} \tag{7}$$

Define the output as $Y_p(k+1|k)$ during k to $k+p$ sampling time, and define the input as U_k during k to $k+p-1$ sampling time.

$$Y_p(k+1|k) \overset{\text{def}}{=} \begin{bmatrix} y_{k+1|k} \\ y_{k+2|k} \\ \vdots \\ y_{k+p|k} \end{bmatrix}_{p\times 1} \qquad U_k \overset{\text{def}}{=} \begin{bmatrix} u_k \\ u_{k+1} \\ \vdots \\ u_{k+p-1} \end{bmatrix}_{p\times 1}$$

To facilitate analysis, the subscripts in the above matrix only represent the number of vectors (or scalars) in the matrix. According to Eq.(7), the recursive expression of the prediction equation is:

$$Y_p(k+1|k) = \mathcal{S}_x x_k + \mathcal{S}_d w_k + \mathcal{S}_u U_k + \mathcal{S}_f v_k \tag{8}$$

where,

$$\mathcal{S}_x = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^p \end{bmatrix}_{p\times 1}, \mathcal{S}_d = \begin{bmatrix} CB_d \\ CAB_d \\ \vdots \\ CA^{p-1}B_d \end{bmatrix}_{p\times 1}, \mathcal{S}_u = \begin{bmatrix} CB & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ CAB & CB & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \mathbf{0} \\ CA^{m-1}B & CA^{m-2}B & \cdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ CA^{p-1}B & CA^{p-2}B & \cdots & \cdots & CB \end{bmatrix}_{p\times p}, \mathcal{S}_f = \begin{bmatrix} I_{n_y\times n_y} \\ I_{n_y\times n_y} \\ \vdots \\ I_{n_y\times n_y} \end{bmatrix}_{p\times 1}$$

3.2 Predictive controller

The predictive control objective function reflects the requirements for system control performance. Usually, the optimization objective function of predictive control is selected as:

$$J(x_k, U_k, m, p) = \left\| \Gamma_y \left(Y_p(k+1|k) - R_{k+1} \right) \right\|^2 + \left\| \Gamma_u U_k \right\|^2 \tag{9}$$

where Γ_u is the weighted constraint matrix for the control input, Γ_y is the weighted constraint matrix for the control error, R_{k+1} is the setpoint during the prediction horizon.

To clarify the calculation form, the auxiliary variables are defined as follows:

$$\rho \overset{\text{def}}{=} \begin{bmatrix} \Gamma_y (Y_p(k+1|k) - R_{k+1}) \\ \Gamma_u U_k \end{bmatrix} \tag{10}$$

Incorporating Eq.(8) into Eq.(10), we can get:

$$\rho = \begin{bmatrix} \Gamma_y (\mathcal{S}_x x_k + \mathcal{S}_d w_k + \mathcal{S}_u U_k + \mathcal{S}_f v_k - R_{k+1}) \\ \Gamma_u U_k \end{bmatrix} = \begin{bmatrix} \Gamma_y \mathcal{S}_u \\ \Gamma_u \end{bmatrix} U_k - \begin{bmatrix} \Gamma_y \overbrace{(R_{k+1} - S_x x_k - S_f v_k - S_d w_k)}^{E_p(k+1|k)} \\ \mathbf{0} \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} \Gamma_y \mathcal{S}_u \\ \Gamma_u \end{bmatrix} U_k}_{Mz} - \underbrace{\begin{bmatrix} \Gamma_y E_p(k+1|k) \\ \mathbf{0} \end{bmatrix}}_{b} = Mz - b \tag{11}$$

Therefore, based on Eq.(11), the predictive control loss function Eq.(9) can be transformed into the following form:

$$\min_z \rho^{\mathrm{T}} \rho \tag{12}$$

Since the used prediction model is linear, the objective function is quadratic and does not consider the constraints in the control horizon, so the analytical solution of the problem can be obtained directly without solving the open-loop optimization problem by numerical methods. It is easy to know the solution of Eq.(12) is:

$$z^* = (M^{\mathrm{T}} M)^{-1} M^{\mathrm{T}} b$$

where $z = U_k, M = \begin{bmatrix} \Gamma_y \mathcal{S}_u & \Gamma_u \end{bmatrix}^{\mathrm{T}}, B = \begin{bmatrix} \Gamma_y E_p(k+1|k) & \mathbf{0} \end{bmatrix}^{\mathrm{T}}$. Put M and b into z^*, then the optimized solutions of the above predictive control are:

$$U_k^* = (\mathcal{S}_u^{\mathrm{T}} \Gamma_y^{\mathrm{T}} \Gamma_y \mathcal{S}_u + \Gamma_u^{\mathrm{T}} \Gamma_u)^{-1} \mathcal{S}_u^{\mathrm{T}} \Gamma_y^{\mathrm{T}} \Gamma_y E_p(k+1|k) \tag{13}$$

Since the state variables cannot be measured, the state estimation value $\hat{x}_k^+$ obtained by Kalman filter should be used as the initial state value of the prediction model, and the recursive relationship of the error is:

$$E_p(k+1|k) = R_{k+1} - \mathcal{S}_x \hat{x}_k^+ - \mathcal{S}_f v_k - \mathcal{S}_d w_k \tag{14}$$

As shown in Eq.(14), the proposed algorithm has the effect of state feedback compensation, due to the introduction of state variable information.

In summary, the basic predictive control law $u_{1_{k-1}}$ in the proposed algorithm is calculated as follows:

$$u_{1_{k-1}} = U_{k-1}^* = K_{\text{mpc}} E_p(k|k-1) \tag{15}$$

Due to the MPC method only use the first output to the real system, the gain matrix can be calculated by $K_{\text{mpc}} = \begin{bmatrix} I_{n_u\times n_u} & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix} (\mathcal{S}_u^{\mathrm{T}} \Gamma_y^{\mathrm{T}} \Gamma_y \mathcal{S}_u + \Gamma_u^{\mathrm{T}} \Gamma_u)^{-1} \mathcal{S}_u^{\mathrm{T}} \Gamma_y^{\mathrm{T}} \Gamma_y$.

3.3 Compensation controller design

Kalman filter is a very practical state estimation method when the key state variables of actual industrial processes cannot be measured in engineering. Usually, Kalman filter is divided into two parts, namely, the state prediction and the state updating. The state prediction part is:

$$\hat{x}_k^- = A\hat{x}_{k-1}^+ + Bu_{k-1} \tag{16}$$

$$P_k^- = AP_{k-1}^+ A^{\mathrm{T}} + Q_{k-1} \tag{17}$$

and the state updating part:

$$K_k = P_k^- C(CP_k^- C^{\mathrm{T}} + G_k)^{-1} \tag{18}$$

$$P_k^+ = (I - K_k C)P_k^- \tag{19}$$

$$\hat{x}_k^+ = \hat{x}_k^- + K_k(y_k - C\hat{x}_k^-) \tag{20}$$

where $\hat{x}_k^-$ stands for the priori estimated state and $\hat{x}_k^+$ is the posterior estimated state. Because the posterior information y_k is added to modify it, the estimated value is closer to the real state variable. K_k is the Kalman gain matrix updated in Eq.(18) at each sampling moment, P_k^+ stands for the transfer matrix of posterior estimation, P_k^- represents priori estimated transfer matrix, G_k is the variance of measuring noise, Q_k is the variance of process noise.

After the un-measurable state variables have been estimated by the Kalman filter, the optimal gain matrix is required to be calculated. Involved in the subsequent analysis, the following assumption which is about the relationship between the variables is given.

Assumption 2. Aim to the system as Eq.(1), the covariance between system state x_k, measuring noise v_k and process noise w_k is zero, which is:

$$P_{xv} = P_{xw} = P_{wv} = \mathbf{0}$$

Remark 5. Since the state variables are affected by noise, the covariance between them is not zero, but in actual analysis, it is found that the covariance between them is much smaller than the variance of each variable itself. Therefore, for the convenience of calculation, it is assumed that they are not related to each other. □

In linear systems, reducing the fluctuation of the error can be achieved by optimizing the variance of the state variable. Given the linear system, this paper just employs a gain matrix $K_{1_{k-1}}$ to optimize the state. Then, putting Eq.(20) into Eq.(4) can get:

$$\begin{aligned} u_{2_{k-1}} &= K_{1_{k-1}} \hat{x}_{k-1}^+ \\ &= K_{1_{k-1}}(I - K_{k-1}C)\hat{x}_{k-1}^- + K_{1_{k-1}} K_{k-1}(Cx_{k-1} + Fv_{k-1}) \end{aligned} \tag{21}$$

Then put $u_{k-1} = u_{1_{k-1}} + u_{2_{k-1}}$ into Eq.(1):

$$\begin{aligned} x_k &= Ax_{k-1} + B(u_{1_{k-1}} + u_{2_{k-1}}) + B_d w_{k-1} \\ &= (A - BK_{\mathrm{mpc}} S_x K_{k-1} C + BK_{1_{k-1}} K_{k-1} C)x_{k-1} + BK_{\mathrm{mpc}} R_k \\ &\quad + B(K_{1_{k-1}} K_{k-1} F - K_{\mathrm{mpc}} S_x K_{k-1} F - K_{\mathrm{mpc}} S_f)v_{k-1} \\ &\quad + (B_d - BK_{\mathrm{mpc}} S_d)w_{k-1} + B(K_{1_{k-1}} - K_{\mathrm{mpc}} S_x)(I - K_{k-1}C)\hat{x}_{k-1}^- \end{aligned} \tag{22}$$

By Assumption 2, the variance between x_{k-1}, v_{k-1} and w_{k-1} is zero, then the relationship between them is:

$$\begin{aligned} P_{x_k} &= D\{(A - BK_{\mathrm{mpc}} S_x K_{k-1} C + BK_{1_{k-1}} K_{k-1} C)x_{k-1} \\ &\quad + BK_{\mathrm{mpc}} R_k + B(K_{1_{k-1}} K_{k-1} F - K_{\mathrm{mpc}} S_x K_{k-1} F - K_{\mathrm{mpc}} S_f)v_{k-1} \\ &\quad + (B_d - K_{\mathrm{mpc}} S_d B)w_{k-1} + B(K_{1_{k-1}} - K_{\mathrm{mpc}} S_x)(I - K_{k-1}C)\hat{x}_{k-1}^-\} \end{aligned} \tag{23}$$

P_{x_k} represents the variance of the variable x_k. $D\{x\}$ represents the calculation of the variance of the variable x. Therefore, the following formula can be obtained by expanding the variance of each variable:

$$P_{x_k} = L_1 P_{x_{k-1}} L_1^{\mathrm{T}} + L_2 D\{\hat{x}_{k-1}^-\} L_2^{\mathrm{T}} + L_3 G_{k-1} L_3^{\mathrm{T}} + L_4 Q_{k-1} L_4^{\mathrm{T}} \tag{24}$$

where L_1, L_2, L_3, and L_4 can be formulated by:

$$\begin{cases} L_1 = A - BK_{\mathrm{mpc}} S_x K_{k-1} C + BK_{1_{k-1}} K_{k-1} C \\ L_2 = B(K_{1_{k-1}} - K_{\mathrm{mpc}} S_x)(I - K_{k-1} C) \\ L_3 = B(K_{1_{k-1}} K_{k-1} F - K_{\mathrm{mpc}} S_x K_{k-1} F - K_{\mathrm{mpc}} S_f) \\ L_4 = B_d - BK_{\mathrm{mpc}} S_d \end{cases}$$

Since it is a linear system and P_{x_k} can be considered as a function of $K_{1_{k-1}}$, the optimal gain matrix $K_{1_{k-1}}$ can be obtained by finding the extreme points of P_{x_k}, as shown below:

$$\begin{aligned} \frac{\partial P_{x_k}}{\partial K_{1_{k-1}}} &= 2B^{\mathrm{T}} C^{\mathrm{T}} K_{k-1}^{\mathrm{T}} P_{x_{k-1}} (A - BK_{\mathrm{mpc}} S_x K_{k-1} C + BK_{1_{k-1}} K_{k-1} C)^{\mathrm{T}} \\ &\quad + 2B^{\mathrm{T}} (I - K_{k-1} C)^{\mathrm{T}} D\{\hat{x}_{k-1}^-\} [K_{1_{k-1}} (I - K_{k-1} C) + K_{\mathrm{mpc}} S_x K_{k-1} C_{k-1} - K_{\mathrm{mpc}} S_x]^{\mathrm{T}} B^{\mathrm{T}} \\ &\quad + 2B^{\mathrm{T}} F^{\mathrm{T}} K_{k-1}^{\mathrm{T}} G_{k-1} (K_{1_{k-1}} K_{k-1} F - K_{\mathrm{mpc}} S_x K_{k-1} F - K_{\mathrm{mpc}} S_f)^{\mathrm{T}} B^{\mathrm{T}} \end{aligned} \tag{25}$$

Then the gain matrix $K_{1_{k-1}}$ can be written as follows:

$$K_{1_{k-1}} = -(M_1 M_2 M_3)^{\mathrm{T}} \tag{26}$$

M_1, M_2, M_3 are calculated by:

$$\begin{cases} M_1 = [C^{\mathrm{T}} K_{k-1}^{\mathrm{T}} P_{x_{k-1}} K_{k-1}^{\mathrm{T}} C^{\mathrm{T}} + (I - K_{k-1}C) D\{\hat{x}_{k-1}^{-}\}(I - K_{k-1}C)^{\mathrm{T}} + K_{k-1} F Q_{k-1} F^{\mathrm{T}} K_{k-1}^{\mathrm{T}}]^{\dagger} \\ M_2 = C^{\mathrm{T}} K_{k-1}^{\mathrm{T}} P_{x_{k-1}} [A^{\mathrm{T}} - (BK_{\mathrm{mpc}} S_x K_{k-1} C)^{\mathrm{T}}] + (I - K_{k-1}C) D\{\hat{x}_{k-1}^{-}\} S_x^{\mathrm{T}} K_{\mathrm{mpc}}^{\mathrm{T}} (C_{k-1}^{\mathrm{T}} K_{k-1}^{\mathrm{T}} - I) B^{\mathrm{T}} \\ \qquad - K_{k-1} F Q_{k-1} [(K_{\mathrm{mpc}} S_x K_{k-1} F)^{\mathrm{T}} + S_f^{\mathrm{T}} K_{\mathrm{mpc}}^{\mathrm{T}}] B^{\mathrm{T}} \\ M_3 = (B^{\mathrm{T}})^{\dagger} \end{cases}$$

In Eq.(26), $(\cdot)^{\dagger}$ is the Moore-Penrose generalized inverse.

Remark 6. When the matrix is not full rank or the matrix is not square, the inverse of the above matrix does not exist. Therefore, the Moore-Penrose generalized inverse[21] is used to solve this problem. □

Notice that the variance $D\{\hat{x}_k^{-}\}$ in M_2 can be updated by recursion method, so put Eqs. (2), (21), (15) into $D(\hat{x}_k^{-})$, we can get the relationship between $D\{\hat{x}_k^{-}\}$ and $\hat{x}_{k-1}^{-}$, x_{k-1}, v_{k-1}, w_{k-1}, as follows:

$$\begin{aligned} D\{\hat{x}_k^{-}\} = D\{ & [A(I - K_{k-1}C) + B(K_{1_{k-1}} - K_{\mathrm{mpc}} S_x)(I - K_{k-1}C)]\hat{x}_{k-1}^{-} \\ & + [K_{k-1}C + B(K_{1_{k-1}} K_{k-1} C - K_{\mathrm{mpc}} S_x K_{k-1} C)] x_{k-1} \\ & + [K_{k-1}F + B(K_{1_{k-1}} K_{k-1} F - K_{\mathrm{mpc}} S_x K_{k-1} F - K_{\mathrm{mpc}} S_f)] v_{k-1} \\ & + BK_{\mathrm{mpc}} R_k - BK_{\mathrm{mpc}} S_d w_{k-1}\} \end{aligned} \tag{27}$$

According to Assumption 2, the covariance between $\hat{x}_{k-1}^{-}$, x_{k-1}, w_{k-1} and v_{k-1} is equal to zero, and the relationship of $D\{\hat{x}_k^{-}\}$ between $D\{\hat{x}_{k-1}^{-}\}, P_{x_{k-1}}, R_{k-1}$ can be obtained by expanding Eq.(27), as shown follows:

$$D\{\hat{x}_k^{-}\} = M_4 D\{\hat{x}_{k-1}^{-}\} M_4^{\mathrm{T}} + M_5 P_{x_{k-1}} M_5^{\mathrm{T}} + M_6 Q_{k-1} M_6^{\mathrm{T}} + M_7 G_{k-1} M_7^{\mathrm{T}} \tag{28}$$

where

$$\begin{cases} M_4 = A(I - K_{k-1}C) + B(K_{1_{k-1}} - K_{\mathrm{mpc}} S_x)(I - K_{k-1}C) \\ M_5 = K_{k-1}C + B(K_{1_{k-1}} K_{k-1} C - K_{\mathrm{mpc}} S_x K_{k-1} C) \\ M_6 = K_{k-1}F + B(K_{1_{k-1}} K_{k-1} F - K_{\mathrm{mpc}} S_x K_{k-1} F - K_{\mathrm{mpc}} S_f) \\ M_7 = BK_{\mathrm{mpc}} S_d \end{cases}$$

Therefore, the gain matrix $K_{1_{k-1}}$ of the Kalman filter compensator can be calculated iteratively through Eq.(26).

In summary, the implementation steps of the proposed En-MPC method are summarized as follows:

Table 1 The implementation steps of En-MPC

Algorithm1: The En-MPC algorithm	
Step 1:	Initialize the system parameters.
Step 2:	Estimate the system states $\hat{x}_{k-1}^{+}$ by Eqs.(14)-(18).
Step 3:	Calculate the basic control input $u_{1_{k-1}}$ by Eq.(3).
Step 4:	Calculate the gain matrix $K_{1_{k-1}}$ by Eq.(26).
Step 5:	Calculate the compensation input $u_{2_{k-1}}$ by Eq.(4).
Step 6:	Calculate the control input u_{k-1} by Eq.(2).
Step 7:	Set $k = k+1$ and go to Step 2.

4 Stability Analysis

MPC obtains an open-loop optimization solution by solving object functions in the finite horizon, but the open-loop optimality does not guarantee the stability of the closed-loop control system. Therefore, the stability of the proposed enhanced control method needs to be analyzed. The objective function is as follows:

$$J(x_k, U_k, m, p) = \left\| \Gamma_y (Y_p(k+1|k) - R_{k+1}) \right\|^2 + \left\| \Gamma_u U_k \right\|^2 \tag{29}$$

For the optimization problem shown in Eq. (29), there will be an optimized value J_k^* at sampling time k, and there will be an optimized value J_{k+1}^* at sampling time $k+1$, but the optimized calculation at sampling time $k+1$ does not guarantee the function value decrease, so the value of the objective function of the next time may appear greater than the previous moment. This will cause the output of the system to tend to infinity in the future. In response to this problem, the work in [22, 23] summarized predictive control in the infinite horizon, that is, extending the finite horizon prediction to infinite to obtain the stability of the system, but this cannot be achieved in actual engineering. The work in [24] proposes a method for constraining the terminal state to ensure the stability of the system, but this will greatly increase the number of online calculations.

By adopting the state-space prediction model and selecting the quadratic objective function, the analytical solution of the open-loop optimization problem can be obtained without numerical analysis. About the stability analysis of the proposed control algorithm, it is considered to prove the upper bounds of the system states by recursively analyzing the state-space functions, since the proposed control method only adds compensation on the basic control input. To prove the stability of the system, the assumption is given as follows:

Assumption 3. For the system shown in Eq.(1), $x_{k\text{-}1}$ is the state of the system at sampling time $k-1$, and its estimated value is $\hat{x}_{k\text{-}1}^{-}$, and there is a constant δ that satisfies the following inequality relationship:

$$\left\|\hat{x}_{k\text{-}1}^{-}\right\| \le \delta^2 \left\|x_{k\text{-}1}\right\| \tag{30}$$

Remark 7. $\hat{x}_{k\text{-}1}^{-}$ is the posterior estimated value of the state by the Kalman filter technique, and $x_{k\text{-}1}$ is the state value. In the range of reasonable estimation error, there exists a constant δ such that the respective two norms satisfy the above formula. □

To facilitate the analysis of the boundedness of state variables, we first make the following transformations to the state equation. Integrating the same variables in Eq.(22), the relationship between x_k, $x_{k\text{-}1}$, $\hat{x}_{k\text{-}1}^{-}$ and $\tilde{N}_{k\text{-}1}$ can be established, as shown follows:

$$x_k = \tilde{H}_{k\text{-}1}x_{k\text{-}1} + \tilde{G}_{k\text{-}1}\hat{x}_{k\text{-}1}^{-} + \tilde{N}_{k\text{-}1} \tag{31}$$

where,

$$\begin{cases} \tilde{H}_{k\text{-}1} = A - BK_{\text{mpc}}S_xK_{k\text{-}1}C + BK_{1_{k\text{-}1}}K_{k\text{-}1}C \\ \tilde{G}_{k\text{-}1} = B(K_{\text{mpc}}S_xK_{k\text{-}1}C_{k\text{-}1} + K_{1_{k\text{-}1}}(I - K_{k\text{-}1}C) - K_{\text{mpc}}S_x) \\ \tilde{N}_{k\text{-}1} = BK_{\text{mpc}}R_k + \tilde{F}_{k\text{-}1}v_{k\text{-}1} + \tilde{M}_{k\text{-}1}d_{k\text{-}1} \\ \tilde{F}_{k\text{-}1} = B(K_{1_{k\text{-}1}}K_{k\text{-}1}F - K_{\text{mpc}}S_xK_{k\text{-}1}F - K_{\text{mpc}}S_f) \\ \tilde{M}_{k\text{-}1} = (B_d - BK_{\text{mpc}}S_d) \end{cases} \tag{32}$$

The theorem to ensure the stability of the proposed algorithm is given below. Under this theorem, the system state variables are bounded under the mean square value.

Theorem 1: *Supposing that the Assumption 3 holds, for the system shown in Eq.(1), considering the system parameters* $\tilde{H}_k$, $\tilde{G}_k$, $\tilde{N}_k$ *and* x_0, *if there exists constants* $0<\eta<1$ *and* $\varsigma>0$ *which ensure the following relationship:*

$$\left\|\tilde{H}_k\right\| + \delta\left\|\tilde{G}_k\right\| = \eta,\quad E\{\left\|\tilde{N}_k\right\|\} \le (1-\eta)^2\varsigma^2,\quad E\{\left\|x_0\right\|^2\} \le \varsigma^2 \tag{33}$$

It can be obtained that the system states are ultimately bounded in the mean-square sense under the En-MPC algorithm which is described by Eqs.(2)-(4). □

Proof: *The boundedness of the state variable will be analyzed under the root mean square by induction. Substitute Eq.(32) into* $E\{x_1^{\mathrm{T}}x_1\}$, *and expand it to get an upper bound by Assumption 3, as shown below:*

$$\begin{aligned} E\{x_1^{\mathrm{T}}x_1\} &= E\{(\tilde{H}_0x_0 + \tilde{G}_0\hat{x}_0^{-})^{\mathrm{T}}(\tilde{H}_0x_0 + \tilde{G}_0\hat{x}_0^{-}) + 2(\tilde{H}_0x_0 + \tilde{G}_0\hat{x}_0^{-})^{\mathrm{T}}\tilde{N}_0 + \tilde{N}_0^{\mathrm{T}}\tilde{N}_0\} \\ &\le (\left\|\tilde{H}_0\right\| + \delta\left\|\tilde{G}_0\right\|)^2 E\{\left\|x_0\right\|^2\} + 2(\left\|\tilde{H}_0\right\| + \delta\left\|\tilde{G}_0\right\|)E\{\left\|x_0\right\|\}E\{\left\|\tilde{N}_0\right\|\} + E\{\left\|\tilde{N}_0\right\|^2\} \\ &< \eta^2\varsigma^2 + 2\eta(1-\eta)\varsigma^2 + (1-\eta)^2\varsigma^2 \\ &= \varsigma^2 \end{aligned} \tag{34}$$

According to the above formula, it can be concluded that there is a constant β_1 *such that* $E\{x_1^{\mathrm{T}}x_1\} \le \beta_1^2\varsigma^2$. *In the same way, the following inequality can be obtained for:*

$$\begin{aligned} E\{x_2^{\mathrm{T}}x_2\} &\le (\left\|\tilde{H}_1\right\| + \delta\left\|\tilde{G}_1\right\|)^2 E\{\left\|x_1\right\|^2\} + E\{\left\|\tilde{N}_1\right\|^2\} \\ &\quad + 2(\left\|\tilde{H}_1\right\| + \delta\left\|\tilde{G}_1\right\|)E\{\left\|x_1\right\|\}E\{\left\|\tilde{N}_1\right\|\} \\ &< \eta^2\beta_1^2\varsigma^2 + 2\beta_1\eta(1-\eta)\varsigma^2 + (1-\eta)^2\varsigma^2 \\ &= (\eta\beta_1 + (1-\eta))^2\varsigma^2 \end{aligned} \tag{35}$$

Hence, there is a constant β_2 *that makes the following formula true:*

$$E\{x_2^{\mathrm{T}}x_2\} \le \beta_2^2\left(\eta\beta_1 + (1-\eta)\right)^2\varsigma^2 \tag{36}$$

Aim to $E\{x_3^{\mathrm{T}}x_3\}$, *it has the following relation:*

$$E\{x_3^{\mathrm{T}}x_3\} \le (\|\tilde{H}_2\| + \delta\|\tilde{G}_2\|)^2 E\{\|x_2\|^2\} + 2(\|\tilde{H}_2\| + \delta\|\tilde{G}_2\|)E\{\|x_2\|\}E\{\|\tilde{N}_2\|\} + E\{\|\tilde{N}_2\|^2\}$$
$$< \eta^2\beta_2^2\left(\eta\beta_1 + (1-\eta)\right)^2\varsigma^2 + 2\beta_2\left(\eta\beta_1 + (1-\eta)\right)\eta(1-\eta)\varsigma^2 + (1-\eta)^2\varsigma^2 \tag{37}$$
$$= \left(\eta\beta_2\left(\eta\beta_1 + (1-\eta)\right) + (1-\eta)\right)^2\varsigma^2$$

Based on the above equations, $E\{x_3^{\mathrm{T}}x_3\}$ *can be re-written as:*

$$E\{x_3^{\mathrm{T}}x_3\} \le \beta_3^2\left(\eta\beta_2\left(\eta\beta_1 + (1-\eta)\right) + (1-\eta)\right)^2\varsigma^2 = \beta_3^2\left(\eta^2\beta_2\beta_1 + \eta\beta_2(1-\eta) + (1-\eta)\right)^2\varsigma^2 \tag{38}$$

Define $H_k^2 = (\sum_{i=1}^{k-1}\tilde{\beta}_{ki} - \eta\sum_{i=1}^{k-2}\tilde{\beta}_{ki} + 1 - \eta)^2$ *, where* $\tilde{\beta}_{ki} = \eta^i\prod_{n=1}^{i}\beta_{k-n}, 0 < \beta_k < 1, k \ge 2$ *. Aim to the sampling time* $k \ge 2$ *, it can be obtained by recursion:*

$$E\{x_k^{\mathrm{T}}x_k\} \le \beta_k^2 H_k^2\varsigma^2 \tag{39}$$

Therefore, the mean square Eq.(40) is obtained at sampling time $k+1$ *:*

$$E\{x_{k+1}^{\mathrm{T}}x_{k+1}\} \le (\|\tilde{H}_k\| + \delta\|\tilde{G}_k\|)^2 E\{\|x_k\|^2\} + E\{\|\tilde{N}_k\|^2\}$$
$$+ 2(\|\tilde{H}_k\| + \delta\|\tilde{G}_k\|)E\{\|x_k\|\}E\{\|\tilde{N}_k\|\} \tag{40}$$
$$< \eta^2\beta_k^2 H_k^2\varsigma^2 + (1-\eta)^2\varsigma^2 + 2\beta_k H_k\eta(1-\eta)\varsigma^2$$
$$= \varsigma^2(\eta\beta_k H_k + 1 - \eta)^2$$

When the mean square of state satisfies Eq.(39) at k sampling time, then at k+1 sampling time the mean square of state also satisfies the following inequality relationship:

$$E\{x_{k+1}^{\mathrm{T}}x_{k+1}\} \le \beta_{k+1}^2(\eta\beta_k H_k + (1-\eta))^2\varsigma^2 = \beta_{k+1}^2 H_{k+1}^2\varsigma^2 \tag{41}$$

For the proposed algorithm, under the condition of satisfying Theorem 1, there is always an upper bound of the system state variables at any time, which ensures the stability of the system.

5 Numerical Simulation

To reduce the control error fluctuations and improve control accuracy, the proposed enhanced model predictive control (En-MPC) method mainly suppresses process uncertain dynamics and disturbances through compensation. To fully verify the effectiveness and advancement of the proposed method, numerical simulation verification and sewage treatment process data verification are carried out. First, consider the 2-input 2-output system described by the following state-space model:

$$\begin{cases} x_k = \begin{bmatrix} 0 & 1.4 \\ -0.5 & -0.6 \end{bmatrix} x_{k\text{-}1} + \begin{bmatrix} 0.1 & 0.2 \\ 0.91 & 0.4 \end{bmatrix} u_{k\text{-}1} + w_k \\ y_k = \begin{bmatrix} 0.4 & 0.3 \\ 0.9 & -0.27 \end{bmatrix} x_k + v_k \end{cases} \tag{42}$$

where x_k is the system state, y_k stands for the system output, w_k represents the process noise, and v_k is the measurement noise.

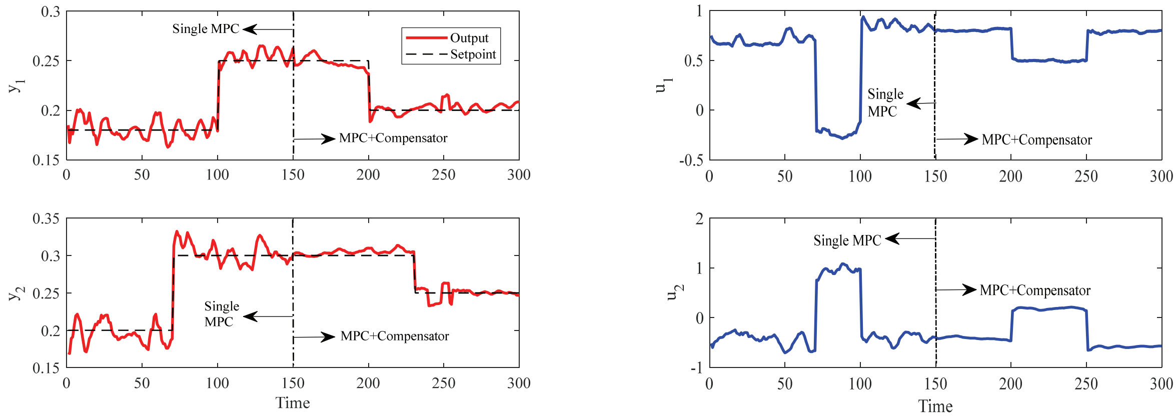

Fig. 2 Setpoint tracking control results with or without proposed control compensation

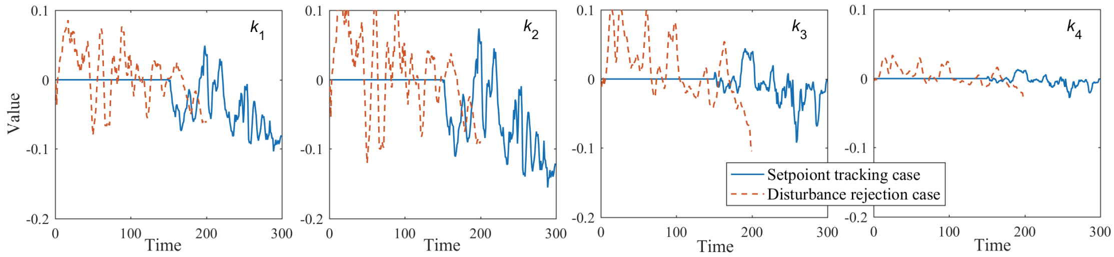

Fig.3 The evolution trend of each element in gain matrix $K_{1_{k-1}}$ of compensation controller in different control experiments

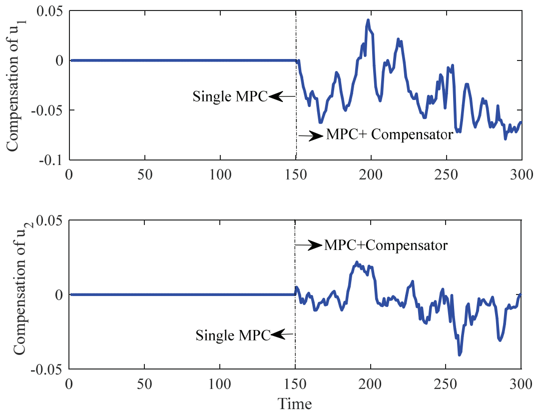

Fig.4 Compensation input

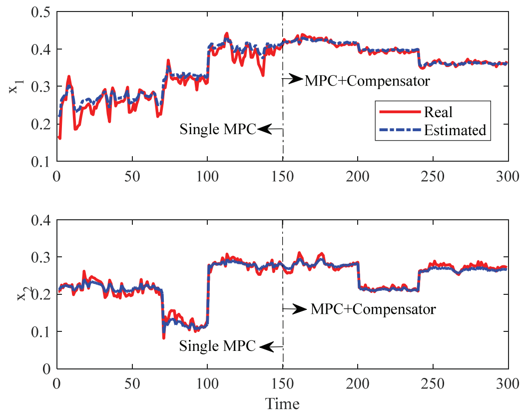

Fig.5 State variable and its estimation

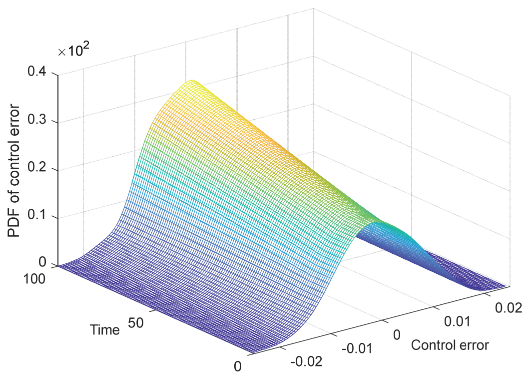

Fig.6 The evolution of error PDF of output y_1

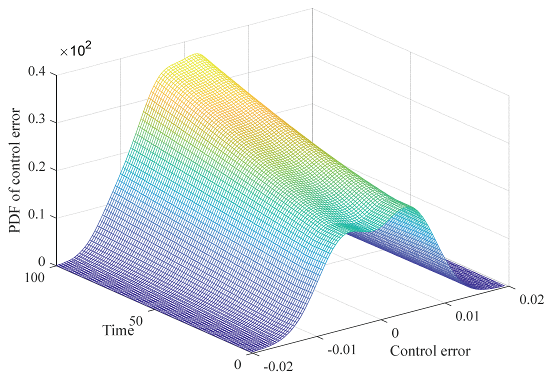

Fig.7 The evolution of error PDF of output y_2

5.1 Setpoint tracking control experiment under non-Gaussian noise

During the experiment, the prediction horizon is $p = 4$, and the control horizon is $m = 4$. Besides, the PDF distributions of process noise and measuring noise are $w_k \sim N(0,0.01)$ and $\gamma_v \sim 0.4N(0,0.01) + 0.6U(-0.02,0.02)$, respectively, and $U(.)$ is the uniform distribution. Besides, the initial value of the state variable is $x_0 = [0.1 \ \ 0.2]^T$, the initial value of the predictive controller output is $u_{1_0} = [0.7 \ \ -0.5]^{\mathrm{T}}$, and the initial value of the compensator output is $u_{2_0} = [0 \ \ 0]^{\mathrm{T}}$. According to Eqs.(13) and (15), the gain matrix K_{mpc} can be obtained as:

$$K_{\mathrm{mpc}} = \begin{bmatrix} 0.0683 & -0.0804 & 0.1171 & 0.3357 & -0.0459 & -0.0705 & -0.0017 & -0.0596 \\ 0.0858 & 0.1036 & 0.0417 & 0.1422 & -0.0286 & -0.0561 & 0.0050 & -0.0206 \end{bmatrix}$$

For comparison, the first 150 moments in the experiment are only using the basic MPC method, and the compensator of the proposed En-MPC method is added to the MPC after 150 moments.

Fig.2 shows the setpoint tracking curves with different control algorithms under the above non-Gaussian noise. Aim to the conventional MPC algorithm, it can track the setpoint, but the value of output and input fluctuate greatly which leads to a larger control error when the system exists noise. Since the proposed En-MPC algorithm uses a control input compensation mechanism, its control output is relatively stable, and the fluctuation of control error is significantly smaller than that of the conventional MPC algorithm. This can reduce the wear of the actuator in practical applications and maintain the stability of the controlled plant.

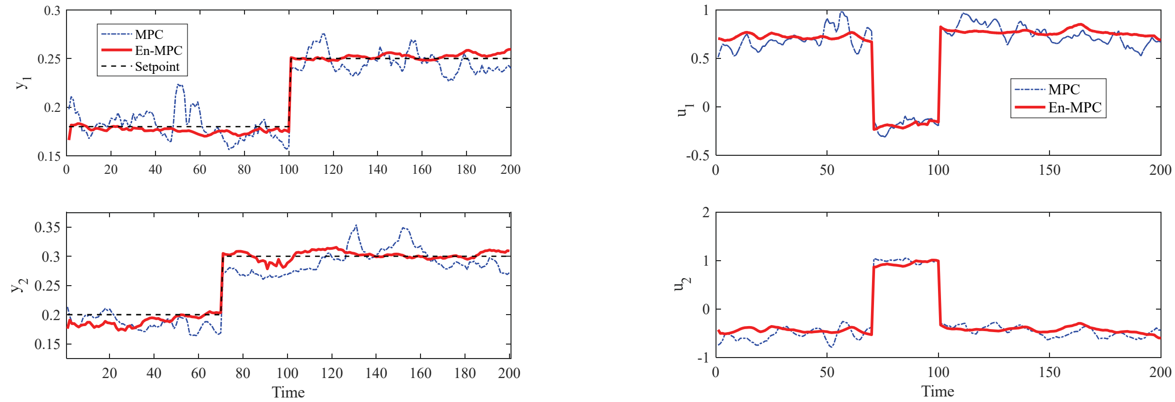

Fig.8 Control results by different control methods under square wave disturbance

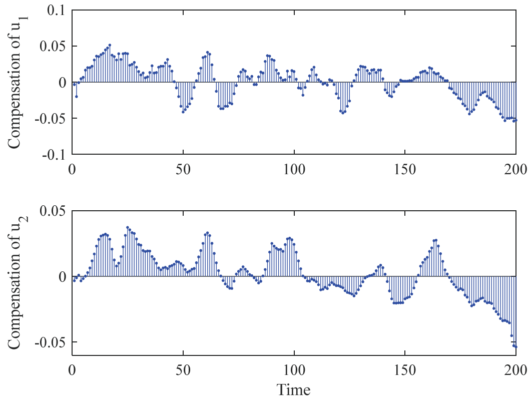

Fig.9 Compensation input

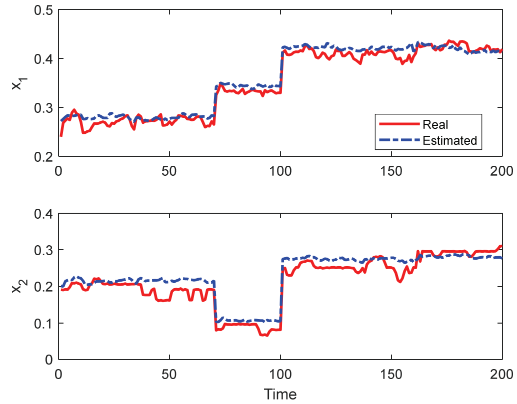

Fig.10 State variable and its estimation

The evolution trend of the each element in the gain matrix $K_{1_{k-1}}$ of designed compensation controller, described in Eq.(26), is shown as solid line in Fig.3, where k_1 in Fig.3 is the first element of $K_{1_{k-1}}$, and the rest is the same. After 150 hours, it can be seen that the gain matrix evolves slowly at each moment to offset the influence of noise. Fig.4 is the compensation input curve of the proposed En-MPC algorithm. It can see that the compensation input maintains a reasonable fluctuation within a certain range after 150 hours, and there is no excessive impact input to affect the system stability. Since the fluctuation of the state is closely related to the output, the curve of the control state variable is drawn in Fig.5. It is found that the fluctuation of the state variable is significantly reduced after the compensation input is added, which is consistent with the reduction of the output fluctuation in Fig.2. At the same time, the dotted line in Fig. 5 is the value of the state estimation obtained by using the Kalman filter technique. The Kalman filter effectively predicts the fluctuation of the state variable which ensures the effectiveness of the compensation input. Fig.6 and Fig.7 respectively show the evolution of the tracking error PDF curves of y_1 and y_2 after adding compensation. The error PDF distribution has become taller and sharper after adding compensation. Therefore, the addition of the compensator effectively reduces the fluctuation of the control error and improves the control accuracy.

5.2 Disturbance rejection experiment under non-Gaussian noise

Under the above-mentioned non-Gaussian noise conditions, square wave disturbance was further added to the control output to test the disturbance rejection performance of the proposed method. Specifically, a square wave disturbance with an amplitude of 0.02 and a period of 20 is added to the output of y_1, and a square wave disturbance with an amplitude of 0.03 and a period of 20 is added to y_2. The other parameter settings in the experiment are consistent with the above experiment, and the results are shown in Figs.8-10.

Fig.8 is the control effect curves of the two algorithms under noise and square wave disturbance. It can be seen that the conventional MPC method can achieve approximate tracking of the setpoint, but the output and the input fluctuate greatly, that is the control error is very large. Under the proposed En-MPC method which adds the proper compensation to the input, the output of the system is close to the setpoint, with small control errors, and is very smooth and stable. Fig.9 is the compensation input curve of the proposed En-MPC method, and Fig.10 is the corresponding state variable and its estimated curve. Even under square wave disturbance, the proposed method achieves precise estimation of the state which ensures the effectiveness of the compensation. As shown by the dotted line in Fig.3 is the evolution curve of each element in the gain matrix $K_{1_{k-1}}$ of designed compensation controller in this experiment. It can be seen that after adding the output disturbance, the gain matrix evolves slowly at each moment which is correspond to Figs.9-10.

In summary, although under the dual influence of noise and output disturbance, the control output of the system can remain stable and smooth, with only small, low-frequency fluctuations near the setpoint by the proposed En-MPC method. At the same time, the control input adjustment of the proposed method is also stable and smooth, which is beneficial to maintain the stability of the system. Therefore, the proposed method that uses the Kalman filter to accurately estimate the unknown state and uses the compensation to effectively reduce the control error can improve the performance of the setpoint tracking and disturbance rejection of the original single predictive control method.

6 Sewage treatment process data verification

The activated sludge method is the most widely used process in the wastewater treatment. As shown in Fig.11, this process mainly uses the microbial population in the activated sludge to absorb, oxidize, and decompose the organic matter in the sewage, and then remove the nitrogen and phosphorus pollutants through nitrification, denitrification, phosphorus release and absorption. It mainly includes four stages: the primary treatment uses physical methods such as grids and grit tanks to filter larger particulate pollutants and preliminary purification. The secondary treatment uses biochemical reactions to remove soluble organic pollutants, sulfides, etc. The tertiary treatment uses an activated carbon filtration method to purify organic pollutants. Finally, the remaining sludge impurities are treated through the processes of concentration and nitrification and then recycled.

As shown in Fig.11, the denitrification-nitrification process in the biochemical tank is the core process of the activated sludge method, which determines the quality of the final effluent of the sewage treatment. The quality of the denitrification-nitrification reaction process depends on the two key variables of the sewage treatment, which are the nitrate-nitrogen concentration $D_{NO,2}$ in the second zone of the biochemical pond and the dissolved oxygen concentration $D_{O,5}$ in the fifth zone. Aim to these two variables, a higher $D_{NO,2}$ will damage the denitrification environment and increase the nitrate-nitrogen content in the effluent. On the contrary, it will slow down the denitrification process and reduce the total nitrogen removal rate. In addition, too high $D_{O,5}$ will destroy the flocculation of microorganisms in the activated sludge and increase energy consumption, while too low $D_{O,5}$ will affect the decomposition effect of activated sludge on organic matter and increase sludge expansion. Therefore, achieving high-performance control of $D_{NO,2}$ and $D_{O,5}$ is the key to ensuring the stable operation of the sewage treatment process and the quality of the effluent. In actual production, $D_{O,5}$ is usually controlled by adjusting the five-zone dissolved oxygen conversion coefficient $K_{La,5}$ which is adjusted by a blower, and $D_{NO,2}$ is controlled by adjusting the internal flow Q_a which is adjusted by the reflux pump. However, due to the uncertainty and coupling of the biochemical reaction process, as well as the random and non-stationary fluctuations of some factors, the traditional PID control method has a poor control effect, while the work in [25] uses neural networks and combined with the multi-gradient method to solve the multi-objective control method is difficult to implement in engineering. The En-MPC method proposed in this paper adds compensation to the control input based on the MPC algorithm to improve the control effect without changing the existing hardware facilities. The calculation amount is low and the stability is guaranteed, so it has better practicability.

First, based on the sewage treatment process data, the subspace identification algorithm in [16] is used to obtain the prediction model of the form described in Eq.(1), where the system matrix parameters are shown in Eq.(43).

$$A=\begin{bmatrix}-0.6348 & 0.7493 & \cdots & -0.8674\\ -0.0676 & -0.4554 & \cdots & 0.4988\\ \vdots & \vdots & \cdots & \vdots\\ -2.8918 & -0.5341 & \cdots & 3.9029\end{bmatrix}_{6\times 6},\ B=\begin{bmatrix}0.1922 & 0.7917\\ 0.8653 & -0.0756\\ \vdots & \vdots\\ 0.4068 & -0.7145\end{bmatrix}_{6\times 2},\ C=\begin{bmatrix}-0.7801 & \cdots & 0.3564\\ -0.1440 & \cdots & -0.8966\end{bmatrix}_{2\times 6},F=B_d=I \quad (43)$$

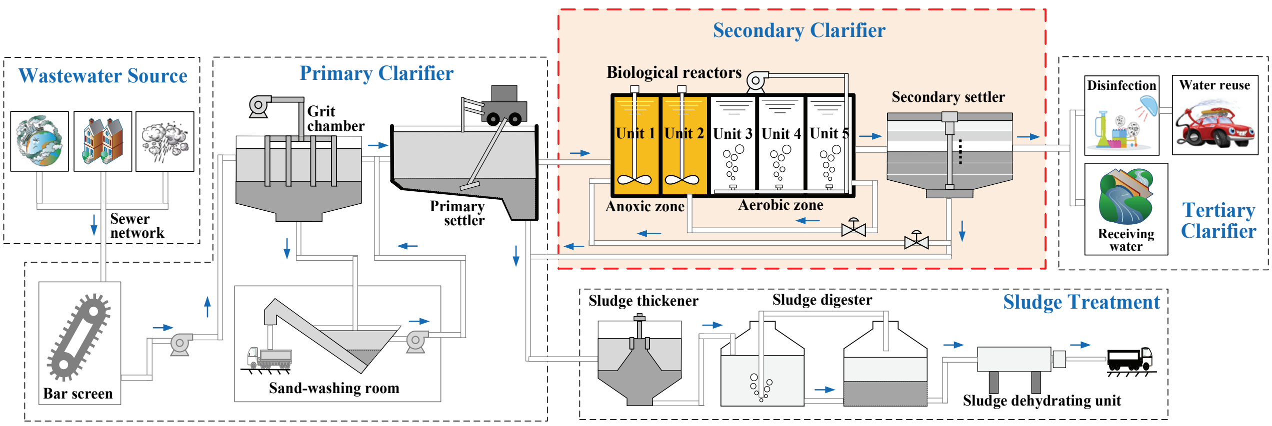

Fig.11 Flow chart of sewage biochemical treatment

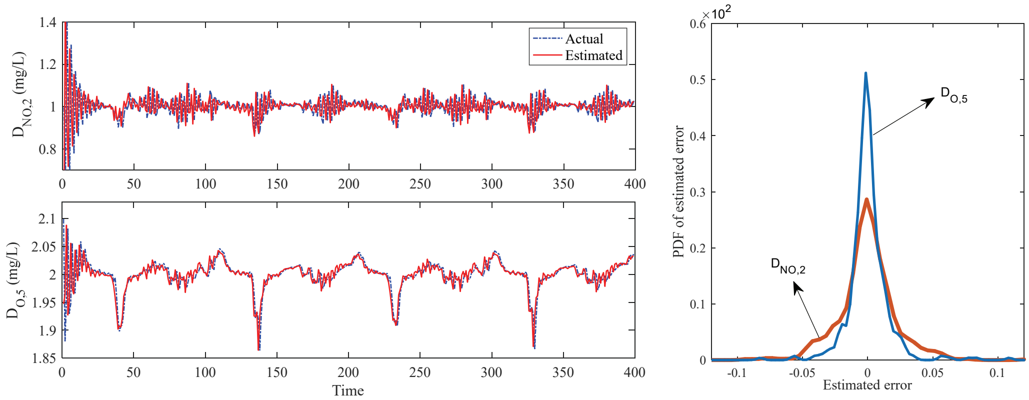

Fig.12 Prediction effects of $S_{NO,2}$ and $D_{O,5}$ by subspace identification algorithm

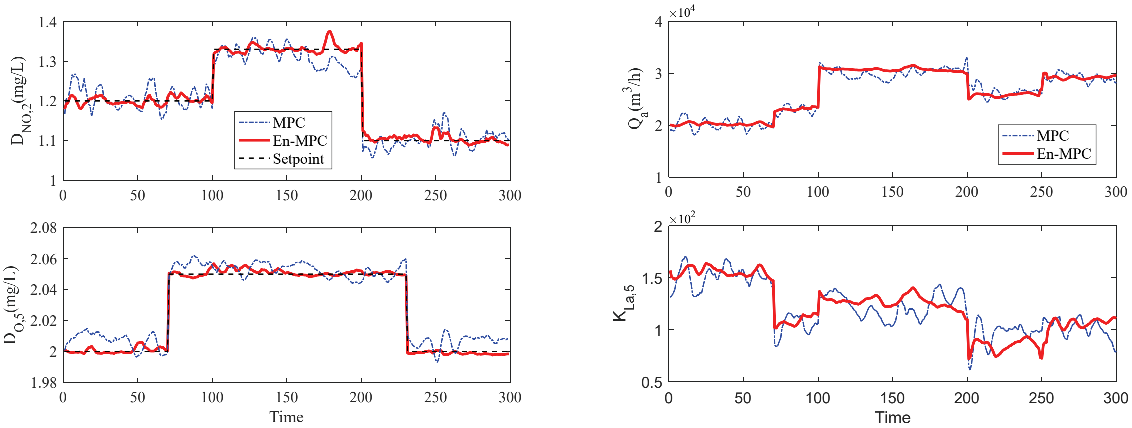

Fig.13 Setpoint tracking control results of $D_{O,5}$ and $D_{NO,2}$ with different control method

As shown in Fig.12, the output of the model can predict the changes of the actual value precisely. The prediction error PDF curves of $D_{NO,2}$ and $D_{O,5}$ also basically obey the high and sharp Gaussian distribution. Therefore, the established model can predict $D_{O,5}$ and $D_{NO,2}$ to meet the precision requirements of predictive control. Then, the proposed En-MPC method is used to control $D_{NO,2}$ and $D_{O,5}$ in the sewage treatment process. The relevant key parameters in the experiment are set as follows: the interchangeable step signals are used as the tracking targets of $D_{NO,2}$ and $D_{O,5}$. The prediction horizon p is 4 and the control horizon m is 4. According to Eqs.(13) and (15), the gain matrix K_{mpc} can be obtained as:

$$K_{\text{mpc}}=\begin{bmatrix} -0.0037 & 0.0010 & 0.0011 & -0.0157 & 0.0223 & 0.0222 & 0.2776 & -0.0967 \\ 0.0015 & -0.0007 & 0.0098 & 0.0041 & -0.0098 & 0.0332 & 0.1536 & -0.0031 \end{bmatrix}$$

A sequence that obeys the distribution of $\gamma_v \sim 0.4N(0,0.03)+0.6U(-0.02,0.02)$ represents the measurement noise that exists in the sewage treatment process. To test the disturbance rejection performance of the proposed method, this paper adds a square wave disturbance with period of 20 and amplitude of 0.05 to $D_{NO,2}$ and a square wave disturbance with period of 20 and amplitude of 0.03 to $D_{O,5}$.

Under the simultaneous action of non-Gaussian noise and output disturbance, the control curve of $D_{NO,2}$ and $D_{O,5}$ are shown as Fig.13, by the proposed En-MPC method and the conventional MPC method. Due to the combined effect of square wave disturbance and non-Gaussian noise, the value of $D_{NO,2}$ and $D_{O,5}$ fluctuates greatly and leads to a large control deviation under the conventional MPC method. It will lead to the unqualified water quality of the final effluent. However, under the proposed En-MPC method, the actual $D_{NO,2}$ and $D_{O,5}$ track the change of the setpoint well through the effective state estimation and control input compensation mechanism. Besides, the two input curves of $K_{La,5}$ and Q_a under the proposed method are much smoother than the conventional MPC method, and the fluctuations are smaller. Fig.14 shows the evolution trend of each element in the gain matrix $K_{1_{k-1}}$ of the compensation controller after unifying and normalizing the data. It can be seen that the gain matrix is still slowly evolving to offset the output disturbance. Fig.15 contains the output curve of the compensator and the state variable estimation curve which is correspond to Fig.14. For the estimation of the unknown state, the Kalman filter effectively tracks the change of the actual state variable and ensures the effective calculation of the control compensation input. Besides, statistical data shows that the computational complexity of En-MPC has been increasing 10.3% compared with the conventional single MPC method, but the control performance of $D_{NO,2}$ and $D_{O,5}$ has been improved significantly.

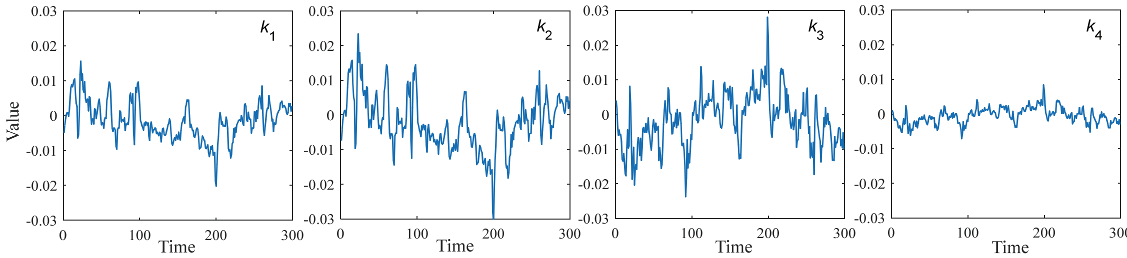

Fig.14 The evolution trend of each element in gain matrix $K_{1_{k-1}}$ of compensation controller

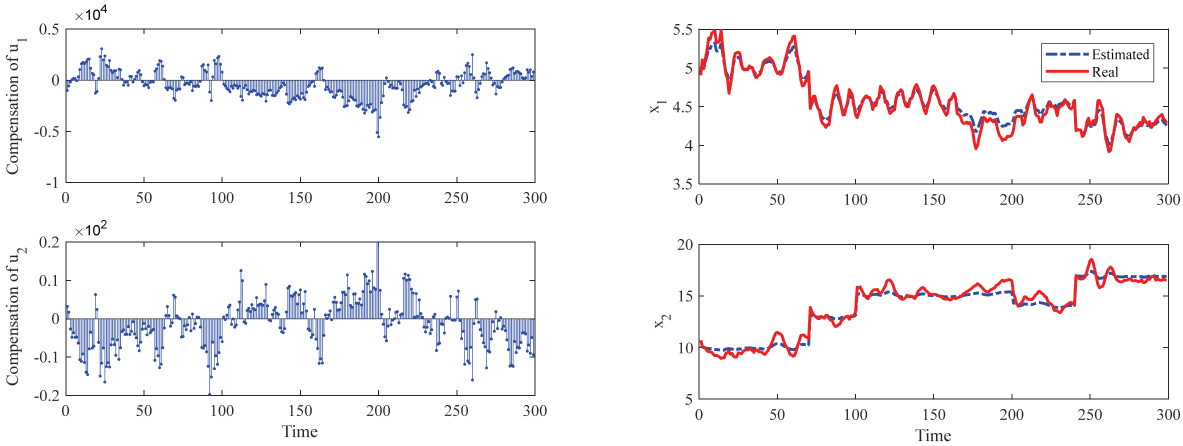

Fig.15 Compensation input (left), state variable and its estimation (right)

7 Conclusion

Aim to the problem of large fluctuations in control errors caused by non-Gaussian noise, model mismatch, external disturbance in the actual industrial plant, this paper proposes a novel enhanced predictive control method. Based on the state-space model predictive control algorithm, it uses Kalman filter to estimate unknown states and combine with the state gain matrix to obtain control compensation. Specifically, the Kalman filter technology is first used to obtain the posterior estimated value of the un-measurable state variable, and then it is multiplied by the gain matrix as the compensation and added with the basic input as the final input to act on the controlled plant to enhance the control performance. The gain matrix is updated iteratively through a recursive relationship based on the variance of noise and state at the current moment. Through inductive reasoning, the upper bound of the system state in the root-mean-square sense is proved, which ensures the stability of the proposed control method. Numerical simulation and sewage treatment process data experiment have verified the effectiveness, advancement, and practicability of the proposed method.

CRediT authorship contribution statement

Xiaoyao Sun: Investigation, Methodology, Validation, Writing-original draf, Softerware. **Ping Zhou**: Conceptualization, Methodology, Writing-review & editing, Formal analysis, Supervision. **Jinliang Ding**: Writing-review & editing, Formal analysis. **Junfei Qiao**: Project administration, Funding acquisition, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Declaration of interests

√ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: